Given: $m_1 = 200 kg$ $r_1 = 0.2 m.$

$m_2 = 300 kg$ $r_2 = 0.15 m$

$m_3 = 240 kg$ $r_3 = 0.25 m$

$m_4 = 260 kg.$ $r_4 = 0.3 m.$

$\theta_1 = 0°$

$\theta_2 = 45°$

$\theta_3 = 45 + 75 = 120°$

$\theta_4 = 45 + 75 + 135° = 255°.$

$r = 0.2 m$; $m = ?$ $\theta = ?$

The magnitude of centrifugal forces are.

$m_1 r_1 = 200 \times 0.2 = 40 kg.m.$

$m_2 r_2 = 300 \times 0.15 = 45 kg.m.$

$m_3 r_3 = 240 \times 0.25 = 60 kg.m.$

$m_4 r_4 = 260 \times 0.3 = 78 kg.m.$

1. Analytical method.

The space diagram is shown in fig.

Resolving $m_1 r_1$, $m_2 r_2$, $m_3 r_3$, $m_4 r_4$ horizontally.

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4.$$

$$\Sigma H = 40 \cos 0^\circ + 45 \cos 45^\circ +$$
$$+ 60 \cos 120^\circ + 78 \cos 255^\circ.$$
$$= 40 + 31.8 - 30 - 20.2$$
$$= 21.6 \text{ Kg. m.}$$

Now, resolving vertically,

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 . r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3$$
$$+ m_4 r_4 \sin\theta_4.$$
$$= 40 \sin 0 + 45 \sin 45^\circ + 60 \sin 120 +$$
$$+ 78 \sin 255^\circ.$$
$$\Sigma V = 0 + 31.8 + 52 - 75.3$$
$$= 8.5 \text{ kg.m.}$$

Resultant $R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$$= \sqrt{21.6^2 + 8.5^2}$$

$$R = 23.2 \text{ kg.m.}$$

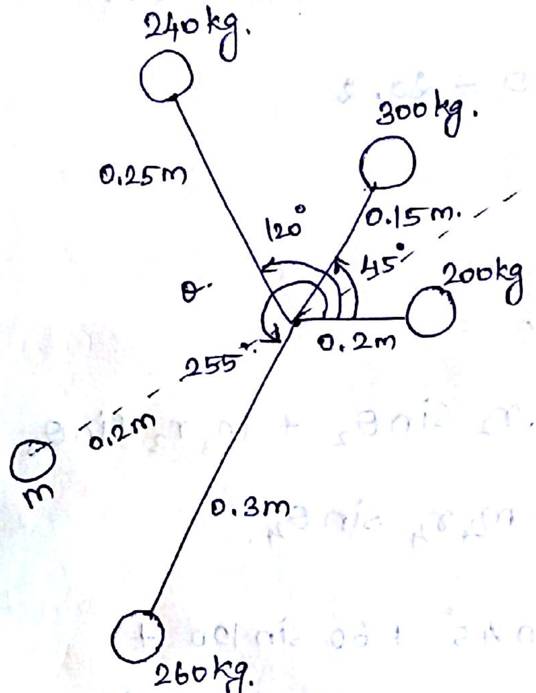
WKT,

$$m.r = R.$$
$$m.r = 23.2 \text{ kg.m}$$
$$m = \frac{23.2}{0.2} = \underline{11.6 \text{ kg}}.$$

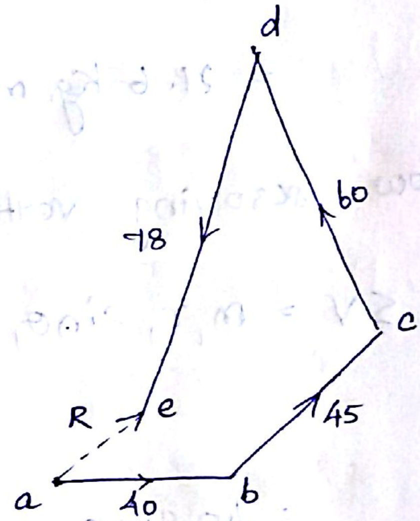
$$\tan\theta' = \frac{\Sigma V}{\Sigma H} = \frac{8.5}{21.6} = 0.393 \Rightarrow \theta' = 21.48^\circ$$

$$\Rightarrow \theta = 180^\circ + 21.48^\circ = \underline{201.48^\circ}.$$

2. Graphical method.



Space diagram.



Vector diagram.

$m \times 0.2 = $ Vector ea.

$= 23$ kg. m.

$m = 115$ kg. ; $\theta = 201°$.

Problem (2)

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg, and 200 kg. respectively. and revolving at radii 80 mm, 70mm, 60mm and 80mm, in planes measured from A at 300mm, 400mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Given :-

$m_A = 200$ kg $r_A = 80$ mm

$m_B = 300$ kg. $r_B = 70$ mm

$m_C = 400$ kg. $r_C = 60$ mm

$m_D = 200$ kg. $r_D = 80$ mm.

$r_X = r_Y = 100$ mm.

m_x = Balancing mass placed in plane X

and

m_y = Balancing mass placed in plane Y.

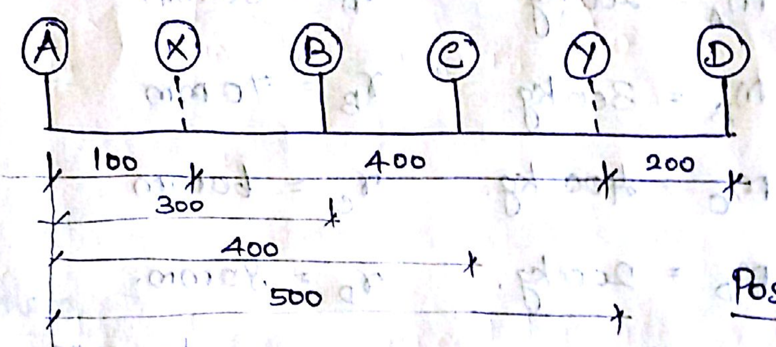
Assume, plane X as Reference plane.

(−)ve ← (X) → (+)ve

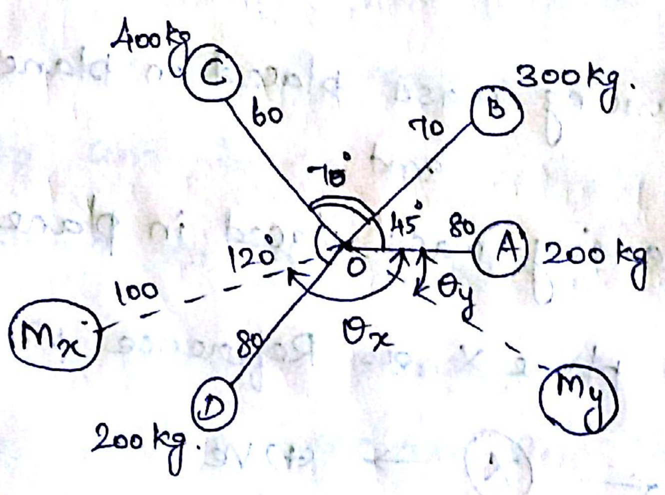
R.P

Plane	Mass (m) kg.	Radius (r) m.	C.F. (m.r) kg.m.	Dist. from R.P. (l) m.	Couple (m.r.l) kg.m^2.
A	200	0.08	16	-0.1	-1.6
(R.P). X	m_x	0.1	$0.1 m_x$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1 m_y$	0.4	$0.04 m_y$
D.	200	0.08	16	0.6	9.6.

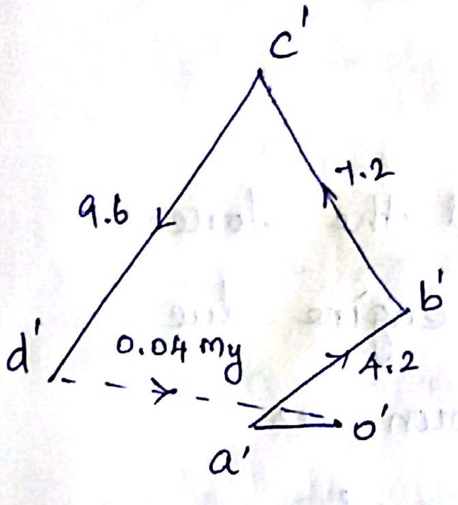
(-)ve. ← R.P → (+)ve.



Position of planes



Angular Position of masses.



Couple polygon.

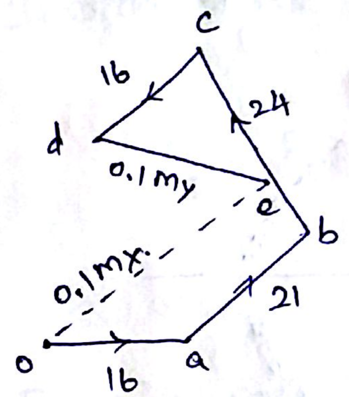
The vector d'o' represents the balanced Couple.

$0.04 m_y$ = Vector d'o'

$= 7.3 \ kg.m^2$.

$\Rightarrow m_y = 182.5 \ kg$.

$\theta_y = 12°$. (clk. from m_A).



Force polygon.

The vector eo represents the Balanced force.

$0.1 m_x$ = Vector eo.

$= 35.5 \ kg.m$.

$m_x = 355 \ kg$.

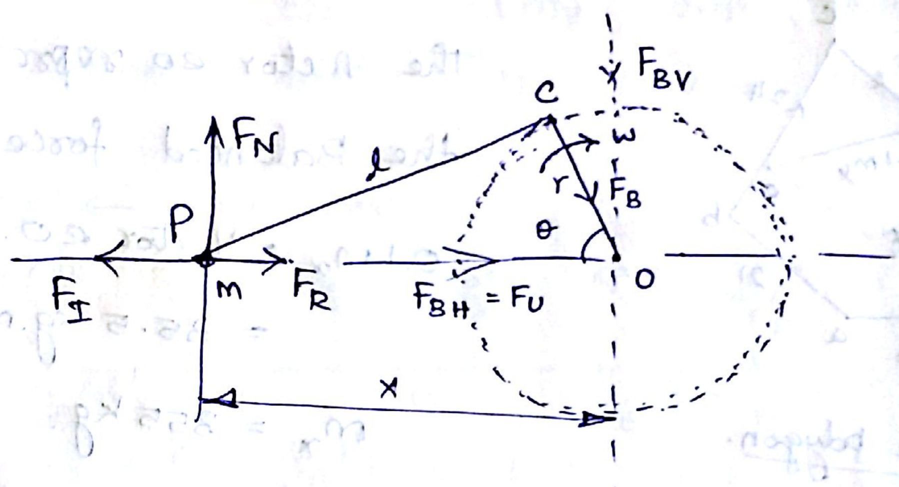
$\theta_x = 145°$ (clk from m_A).

Balancing of reciprocating Masses.

Unbalance force.

The resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalance force or shaking force.

Reciprocating Engine Mechanism.



Let,

F_R = Force required to accelerate the reciprocating parts.

F_I = Inertia force due to reciprocating parts

F_N = Force on the sides of Cylinder walls.

F_B = Force acting on crank shaft Bearing.

m = mass of the reciprocating parts.

l = length of connecting rod PC.

r = radius of the crank OC.

θ = Angle of inclination of the crank with the line of stroke PO.

ω = Angular speed of the crank.

$n = l/r$.

Acceleration of reciprocating parts. a_R.

$$a_R = \omega^2 \cdot r \cdot \left(\cos\theta + \frac{\cos 2\theta}{n}\right).$$

Inertia force due to reciprocating parts.

$$F_I = F_R = \text{mass} \times \text{Acceleration}.$$

$$= m \cdot \omega^2 \cdot r \cdot \left(\cos\theta + \frac{\cos 2\theta}{n}\right).$$

F_{BH} is equal and opposite to inertia force (F_I). This force is an unbalanced one. and is denoted by F_U.

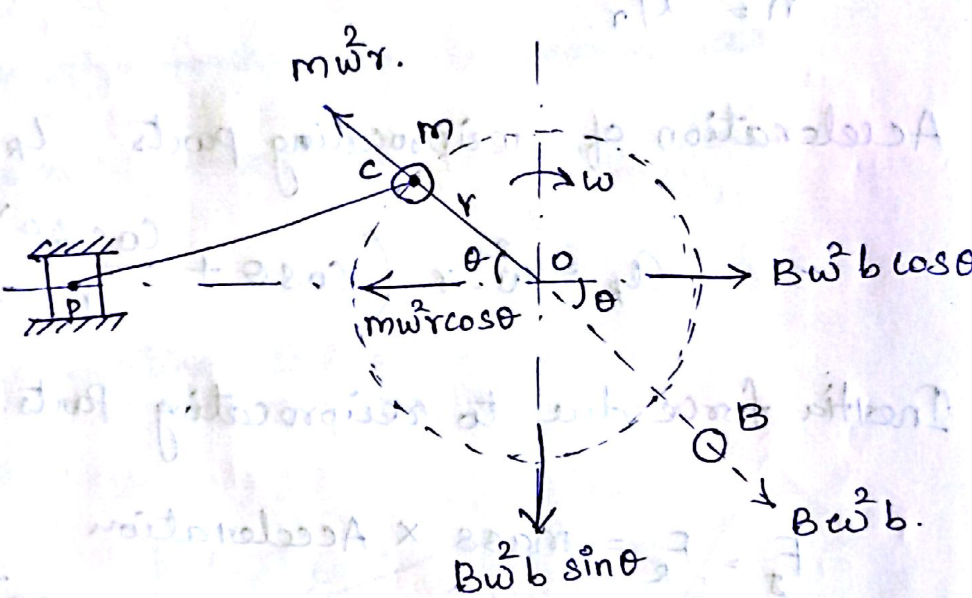
$$F_U = m \cdot \omega^2 \cdot r \cdot \left(\cos\theta + \frac{\cos 2\theta}{n}\right).$$

$$= F_p + F_s.$$

Primary unbalance force $F_p = m\omega^2 r \cdot \cos\theta$.

Secondary unbalance force $F_s = m\omega^2 r \dfrac{\cos 2\theta}{n}$.

Partial Balancing of unbalanced Primary force in a reciprocating Engine.



Centrifugal force due to mass B. $= B.\omega^2.b$.

horizontal Component of this force acting in opposite direction of primary force

$$= B.\omega^2.b.\cos\theta.$$

The primary force is balanced, if,

$$B.\omega^2.b.\cos\theta = m.\omega^2.r.\cos\theta$$

or

$$B.b = m.r.$$

As a Compromise let a fraction 'c' of the reciprocating masses is balanced, Such that $c.m.r = B.b$.

$\therefore$ Unbalance force along the line of stroke,

$$= (1-c)\, m.\omega^2.r.\cos\theta.$$

$\therefore$ Unbalance force along the perpendicular to the line of stroke.

$$= c.m.\omega^2.r.\sin\theta.$$

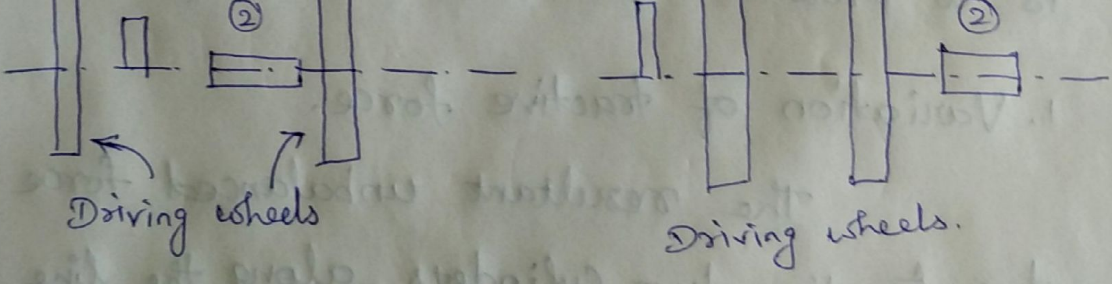
$\therefore$ Resultant unbalanced force at any intant

$$= m.\omega^2.r.\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}.$$

Partial balancing of Locomotives.

The two cylinder locomotives may be classified as.

1. Inside cylinder Locomotives
2. Outside cylinder locomotives.



The locomotives may be.

A single or uncoupled locomotive is one, in which the effort is transmitted to one pair of the wheels only. whereas in coupled locomotives, the driving wheels are

Connected to the leading and trailing wheel by an outside coupling rod.

Effect of Partial balancing of reciprocating Parts of two cylinder locomotives.

The effect of an unbalanced primary force along the line of stroke is to produce;

1. Variation in tractive force along the line of stroke.
2. Swaying couple.

The effect of an unbalanced primary force perpendicular to the line of stroke is to produce ~~variation in~~ Hammer blow.

1. Variation of tractive force.

The resultant unbalanced force due to the two cylinders, along the line of stroke is known as tractive force.